

## Ὅροι.

α'. Ἴσοι κύκλοι εἰσίν, ὧν αἱ διάμετροι ἴσαι εἰσίν, ἢ ὧν αἱ ἐκ τῶν κέντρων ἴσαι εἰσίν.

β'. Εὐθεία κύκλου ἐφάπτεσθαι λέγεται, ἥτις ἀπτομένη τοῦ κύκλου καὶ ἐκβαλλομένη οὐ τέμνει τὸν κύκλον.

γ'. Κύκλοι ἐφάπτεσθαι ἀλλήλων λέγονται οἵτινες ἀπτόμενοι ἀλλήλων οὐ τέμνουσιν ἀλλήλους.

δ'. Ἐν κύκλῳ ἴσον ἀπέχειν ἀπὸ τοῦ κέντρου εὐθεῖαι λέγονται, ὅταν αἱ ἀπὸ τοῦ κέντρου ἐπ' αὐτάς κάθετοι ἀγόμεναι ἴσαι ᾧσιν.

ε'. Μείζων δὲ ἀπέχειν λέγεται, ἐφ' ἣν ἡ μείζων κάθετος πίπτει.

ς'. Τμήμα κύκλου ἐστὶ τὸ περιεχόμενον σχῆμα ὑπὸ τε εὐθείας καὶ κύκλου περιφερείας.

ζ'. Τμήματος δὲ γωνία ἐστὶν ἡ περιεχομένη ὑπὸ τε εὐθείας καὶ κύκλου περιφερείας.

η'. Ἐν τμήματι δὲ γωνία ἐστίν, ὅταν ἐπὶ τῆς περιφερείας τοῦ τμήματος ληφθῆ τι σημεῖον καὶ ἀπ' αὐτοῦ ἐπὶ τὰ πέρατα τῆς εὐθείας, ἢ ἐστὶ βάσις τοῦ τμήματος, ἐπιζευχθῶσιν εὐθεῖαι, ἡ περιεχομένη γωνία ὑπὸ τῶν ἐπιζευχθεισῶν εὐθειῶν.

θ'. Ὅταν δὲ αἱ περιέχουσαι τὴν γωνίαν εὐθεῖαι ἀπολαμβάνωσι τινα περιφέρειαν, ἐπ' ἐκείνης λέγεται βεβηκέναι ἡ γωνία.

ι'. Τομεὺς δὲ κύκλου ἐστίν, ὅταν πρὸς τῷ κέντρῳ τοῦ κύκλου συσταθῆ γωνία, τὸ περιεχόμενον σχῆμα ὑπὸ τε τῶν τὴν γωνίαν περιεχουσῶν εὐθειῶν καὶ τῆς ἀπολαμβανομένης ὑπ' αὐτῶν περιφερείας.

ια'. Ὅμοια τμήματα κύκλων ἐστὶ τὰ δεχόμενα γωνίας ἴσας, ἢ ἐν οἷς αἱ γωνίαι ἴσαι ἀλλήλαις εἰσίν.

α'.

Τοῦ δοθέντος κύκλου τὸ κέντρον εὐρεῖν.

Ἐστω ὁ δοθείς κύκλος ὁ  $ABΓ$ . δεῖ δὴ τοῦ  $ABΓ$  κύκλου τὸ κέντρον εὐρεῖν.

Διήχθω τις εἰς αὐτόν, ὡς ἔτυχεν, εὐθεῖα ἡ  $AB$ , καὶ τετμήσθω δίχα κατὰ τὸ  $\Delta$  σημεῖον, καὶ ἀπὸ τοῦ  $\Delta$  τῆ  $AB$  πρὸς ὀρθὰς ἤχθω ἡ  $\Delta Γ$  καὶ διήχθω ἐπὶ τὸ  $E$ , καὶ τετμήσθω ἡ  $ΓE$  δίχα κατὰ τὸ  $Z$ . λέγω, ὅτι τὸ  $Z$  κέντρον ἐστὶ τοῦ  $ABΓ$  [κύκλου].

Μὴ γάρ, ἀλλ' εἰ δυνατόν, ἔστω τὸ  $H$ , καὶ ἐπεζεύχθωσαν αἱ  $HA$ ,  $H\Delta$ ,  $HB$ . καὶ ἐπεὶ ἴση ἐστὶν ἡ  $A\Delta$  τῆ  $\Delta B$ , κοινὴ δὲ ἡ  $\Delta H$ , δύο δὲ αἱ  $A\Delta$ ,  $\Delta H$  δύο ταῖς  $H\Delta$ ,  $\Delta B$  ἴσαι εἰσίν ἑκατέρᾳ ἑκατέρᾳ· καὶ βάσις ἡ  $HA$  βάσει τῆ  $HB$  ἐστὶν ἴση· ἐκ κέντρου γάρ· γωνία ἄρα ἡ ὑπὸ  $A\Delta H$  γωνία τῆ  $\Delta H B$  ἴση ἐστίν.

## Definitions

1. Equal circles are (circles) whose diameters are equal, or whose (distances) from the centers (to the circumferences) are equal (i.e., whose radii are equal).

2. A straight-line said to touch a circle is any (straight-line) which, meeting the circle and being produced, does not cut the circle.

3. Circles said to touch one another are any (circles) which, meeting one another, do not cut one another.

4. In a circle, straight-lines are said to be equally far from the center when the perpendiculars drawn to them from the center are equal.

5. And (that straight-line) is said to be further (from the center) on which the greater perpendicular falls (from the center).

6. A segment of a circle is the figure contained by a straight-line and a circumference of a circle.

7. And the angle of a segment is that contained by a straight-line and a circumference of a circle.

8. And the angle in a segment is the angle contained by the joined straight-lines, when any point is taken on the circumference of a segment, and straight-lines are joined from it to the ends of the straight-line which is the base of the segment.

9. And when the straight-lines containing an angle cut off some circumference, the angle is said to stand upon that (circumference).

10. And a sector of a circle is the figure contained by the straight-lines surrounding an angle, and the circumference cut off by them, when the angle is constructed at the center of a circle.

11. Similar segments of circles are those accepting equal angles, or in which the angles are equal to one another.

## Proposition 1

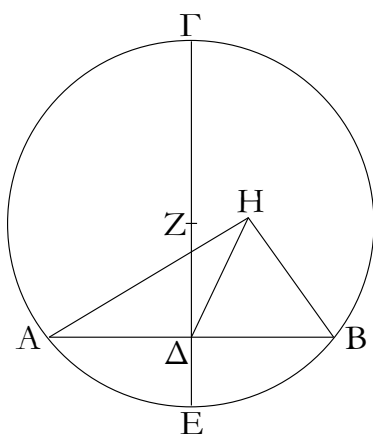
To find the center of a given circle.

Let  $ABC$  be the given circle. So it is required to find the center of circle  $ABC$ .

Let some straight-line  $AB$  have been drawn through ( $ABC$ ), at random, and let ( $AB$ ) have been cut in half at point  $D$  [Prop. 1.9]. And let  $DC$  have been drawn from  $D$ , at right-angles to  $AB$  [Prop. 1.11]. And let ( $CD$ ) have been drawn through to  $E$ . And let  $CE$  have been cut in half at  $F$  [Prop. 1.9]. I say that (point)  $F$  is the center of the [circle]  $ABC$ .

For (if) not then, if possible, let  $G$  (be the center of the circle), and let  $GA$ ,  $GD$ , and  $GB$  have been joined. And since  $AD$  is equal to  $DB$ , and  $DG$  (is) common, the two

ὅταν δὲ εὐθεΐα ἐπ' εὐθεΐαν σταθεῖσα τὰς ἐφεξῆς γωνίας ἴσας ἀλλήλαις ποιῆ, ὀρθὴ ἑκατέρα τῶν ἴσων γωνιῶν ἐστὶν ὀρθή· ἄρα ἐστὶν ἡ ὑπὸ  $H\Delta B$ . ἐστὶ δὲ καὶ ἡ ὑπὸ  $Z\Delta B$  ὀρθή· ἴση ἄρα ἡ ὑπὸ  $Z\Delta B$  τῇ ὑπὸ  $H\Delta B$ , ἡ μείζων τῇ ἐλάττωι· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα τὸ  $H$  κέντρον ἐστὶ τοῦ  $AB\Gamma$  κύκλου. ὁμοίως δὲ δείξομεν, ὅτι οὐδ' ἄλλο τι πλὴν τοῦ  $Z$ .



Τὸ  $Z$  ἄρα σημεῖον κέντρον ἐστὶ τοῦ  $AB\Gamma$  [κύκλου].

Πόρισμα.

Ἐκ δὲ τούτου φανερόν, ὅτι ἐὰν ἐν κύκλῳ εὐθεΐα τις εὐθεϊάν τινα δίχα καὶ πρὸς ὀρθὰς τέμνη, ἐπὶ τῆς τεμνούσης ἐστὶ τὸ κέντρον τοῦ κύκλου. — ὅπερ ἔδει ποιῆσαι.

† The Greek text has “ $GD, DB$ ”, which is obviously a mistake.

β'.

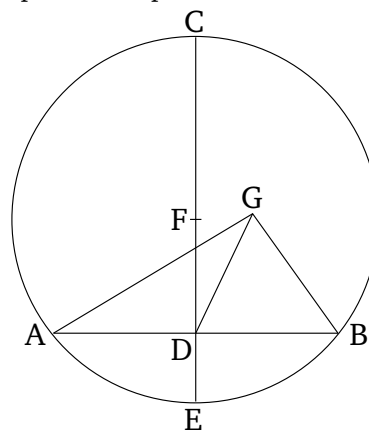
Ἐὰν κύκλου ἐπὶ τῆς περιφερείας ληφθῆ δύο τυχόντα σημεῖα, ἢ ἐπὶ τὰ σημεῖα ἐπιζευγνυμένη εὐθεΐα ἐντὸς πεσεῖται τοῦ κύκλου.

Ἐστω κύκλος ὁ  $AB\Gamma$ , καὶ ἐπὶ τῆς περιφερείας αὐτοῦ εἰληφθῶ δύο τυχόντα σημεῖα τὰ  $A, B$ · λέγω, ὅτι ἡ ἀπὸ τοῦ  $A$  ἐπὶ τὸ  $B$  ἐπιζευγνυμένη εὐθεΐα ἐντὸς πεσεῖται τοῦ κύκλου.

Μὴ γάρ, ἀλλ' εἰ δυνατόν, πιπτέτω ἐκτὸς ὡς ἡ  $AEB$ , καὶ εἰληφθῶ τὸ κέντρον τοῦ  $AB\Gamma$  κύκλου, καὶ ἔστω τὸ  $\Delta$ , καὶ ἐπεζεύχθωσαν αἱ  $\Delta A, \Delta B$ , καὶ διήχθω ἡ  $\Delta ZE$ .

Ἐπεὶ οὖν ἴση ἐστὶν ἡ  $\Delta A$  τῇ  $\Delta B$ , ἴση ἄρα καὶ γωνία ἡ ὑπὸ  $\Delta AE$  τῇ ὑπὸ  $\Delta BE$ · καὶ ἐπεὶ τριγώνου τοῦ  $\Delta AE$  μία

(straight-lines)  $AD, DG$  are equal to the two (straight-lines)  $BD, DG$ ,<sup>†</sup> respectively. And the base  $GA$  is equal to the base  $GB$ . For (they are both) radii. Thus, angle  $ADG$  is equal to angle  $GDB$  [Prop. 1.8]. And when a straight-line stood upon (another) straight-line make adjacent angles (which are) equal to one another, each of the equal angles is a right-angle [Def. 1.10]. Thus,  $GDB$  is a right-angle. And  $FDB$  is also a right-angle. Thus,  $FDB$  (is) equal to  $GDB$ , the greater to the lesser. The very thing is impossible. Thus, (point)  $G$  is not the center of the circle  $ABC$ . So, similarly, we can show that neither is any other (point) except  $F$ .



Thus, point  $F$  is the center of the [circle]  $ABC$ .

Corollary

So, from this, (it is) manifest that if any straight-line in a circle cuts any (other) straight-line in half, and at right-angles, then the center of the circle is on the former (straight-line). — (Which is) the very thing it was required to do.

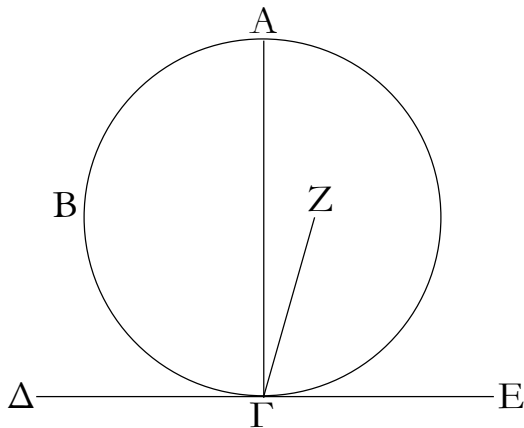
Proposition 2

If two points are taken at random on the circumference of a circle then the straight-line joining the points will fall inside the circle.

Let  $ABC$  be a circle, and let two points  $A$  and  $B$  have been taken at random on its circumference. I say that the straight-line joining  $A$  to  $B$  will fall inside the circle.

For (if) not then, if possible, let it fall outside (the circle), like  $AEB$  (in the figure). And let the center of the circle  $ABC$  have been found [Prop. 3.1], and let it be (at point)  $D$ . And let  $DA$  and  $DB$  have been joined, and let  $DFE$  have been drawn through.

Therefore, since  $DA$  is equal to  $DB$ , the angle  $DAE$

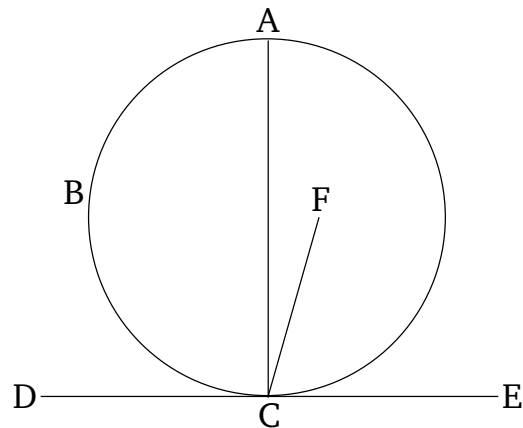


Μή γάρ, ἀλλ' εἰ δυνατόν, ἔστω τὸ Z, καὶ ἐπεζεύχθω ἡ ΓZ.

Ἐπεὶ [οὖν] κύκλου τοῦ ABΓ ἐφάπτεται τις εὐθεΐα ἡ ΔΕ, ἀπὸ δὲ τοῦ κέντρου ἐπὶ τὴν ἀφὴν ἐπέζευκται ἡ ΖΓ, ἡ ΖΓ ἄρα κάθετός ἐστιν ἐπὶ τὴν ΔΕ· ὀρθὴ ἄρα ἐστὶν ἡ ὑπὸ ΖΓΕ. ἐστὶ δὲ καὶ ἡ ὑπὸ ΑΓΕ ὀρθή· ἴση ἄρα ἐστὶν ἡ ὑπὸ ΖΓΕ τῇ ὑπὸ ΑΓΕ ἢ ἐλάττων τῇ μείζονι· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα τὸ Z κέντρον ἐστὶ τοῦ ABΓ κύκλου. ὁμοίως δὲ δεῖξομεν, ὅτι οὐδ' ἄλλο τι πλὴν ἐπὶ τῆς ΑΓ.

Ἐάν ἄρα κύκλου ἐφάπτηται τις εὐθεΐα, ἀπὸ δὲ τῆς ἀφῆς τῆ ἐφαπτομένη πρὸς ὀρθὰς εὐθεΐα γραμμὴ ἀχθῆ, ἐπὶ τῆς ἀχθείσης ἔσται τὸ κέντρον τοῦ κύκλου· ὅπερ ἔδει δεῖξαι.

angles to  $DE$  [Prop. 1.11]. I say that the center of the circle is on  $AC$ .



For (if) not, if possible, let  $F$  be (the center of the circle), and let  $CF$  have been joined.

[Therefore], since some straight-line  $DE$  touches the circle  $ABC$ , and  $FC$  has been joined from the center to the point of contact,  $FC$  is thus perpendicular to  $DE$  [Prop. 3.18]. Thus,  $FCE$  is a right-angle. And  $ACE$  is also a right-angle. Thus,  $FCE$  is equal to  $ACE$ , the lesser to the greater. The very thing is impossible. Thus,  $F$  is not the center of circle  $ABC$ . So, similarly, we can show that neither is any (point) other (than one) on  $AC$ .

Thus, if some straight-line touches a circle, and a straight-line is drawn from the point of contact, at right-angles to the tangent, then the center (of the circle) will be on the (straight-line) so drawn. (Which is) the very thing it was required to show.

κ'.

Ἐν κύκλῳ ἡ πρὸς τῶ κέντρῳ γωνία διπλασίων ἐστὶ τῆς πρὸς τῇ περιφερείᾳ, ὅταν τὴν αὐτὴν περιφέρειαν βάσιν ἔχωσιν αἱ γωνίαί.

Ἐστω κύκλος ὁ ABΓ, καὶ πρὸς μὲν τῶ κέντρῳ αὐτοῦ γωνία ἔστω ἡ ὑπὸ BEΓ, πρὸς δὲ τῇ περιφερείᾳ ἡ ὑπὸ BAΓ, ἐχέτωσαν δὲ τὴν αὐτὴν περιφέρειαν βάσιν τὴν ΒΓ· λέγω, ὅτι διπλασίων ἐστὶν ἡ ὑπὸ BEΓ γωνία τῆς ὑπὸ BAΓ.

Ἐπιζευχθεῖσα γὰρ ἡ ΑΕ διήχθω ἐπὶ τὸ Z.

Ἐπεὶ οὖν ἴση ἐστὶν ἡ ΕΑ τῇ ΕΒ, ἴση καὶ γωνία ἡ ὑπὸ ΕΑΒ τῇ ὑπὸ ΕΒΑ· αἱ ἄρα ὑπὸ ΕΑΒ, ΕΒΑ γωνίαί τῆς ὑπὸ ΕΑΒ διπλασίους εἰσίν. ἴση δὲ ἡ ὑπὸ BEZ ταῖς ὑπὸ ΕΑΒ, ΕΒΑ· καὶ ἡ ὑπὸ BEZ ἄρα τῆς ὑπὸ ΕΑΒ ἐστὶ διπλῆ. διὰ τὰ αὐτὰ δὲ καὶ ἡ ὑπὸ ZEG τῆς ὑπὸ ΕΑΓ ἐστὶ διπλῆ. ὅλη ἄρα ἡ ὑπὸ BEΓ ὅλης τῆς ὑπὸ BAΓ ἐστὶ διπλῆ.

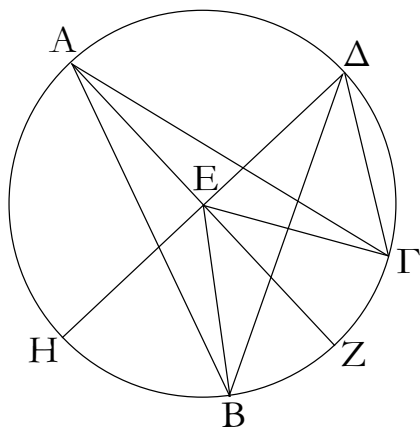
### Proposition 20

In a circle, the angle at the center is double that at the circumference, when the angles have the same circumference base.

Let  $ABC$  be a circle, and let  $BEC$  be an angle at its center, and  $BAC$  (one) at (its) circumference. And let them have the same circumference base  $BC$ . I say that angle  $BEC$  is double (angle)  $BAC$ .

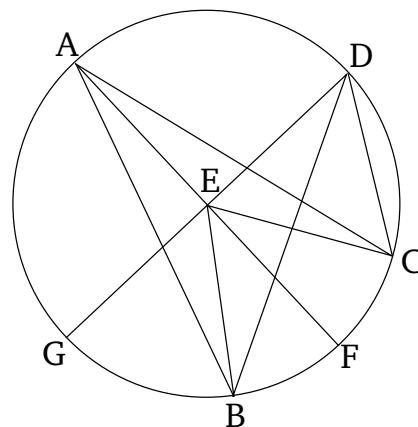
For being joined, let  $AE$  have been drawn through to  $F$ .

Therefore, since  $EA$  is equal to  $EB$ , angle  $EAB$  (is) also equal to  $EBA$  [Prop. 1.5]. Thus, angle  $EAB$  and  $EBA$  is double (angle)  $EAB$ . And  $BEF$  (is) equal to  $EAB$  and  $EBA$  [Prop. 1.32]. Thus,  $BEF$  is also double  $EAB$ . So, for the same (reasons),  $FEC$  is also double  $EAC$ . Thus, the whole (angle)  $BEC$  is double the whole (angle)  $BAC$ .



Κεκλάσθω δὴ πάλιν, καὶ ἔστω ἑτέρα γωνία ἢ ὑπὸ  $B\Delta\Gamma$ , καὶ ἐπιζευχθεῖσα ἡ  $\Delta E$  ἐκβεβλήσθω ἐπὶ τὸ  $H$ . ὁμοίως δὴ δείξομεν, ὅτι διπλῆ ἔστιν ἡ ὑπὸ  $HE\Gamma$  γωνία τῆς ὑπὸ  $E\Delta\Gamma$ , ὧν ἡ ὑπὸ  $HEB$  διπλῆ ἔστι τῆς ὑπὸ  $E\Delta B$ . λοιπὴ ἄρα ἡ ὑπὸ  $BEG$  διπλῆ ἔστι τῆς ὑπὸ  $B\Delta\Gamma$ .

Ἐν κύκλῳ ἄρα ἡ πρὸς τῷ κέντρῳ γωνία διπλασίον ἐστὶ τῆς πρὸς τῇ περιφερείᾳ, ὅταν τὴν αὐτὴν περιφέρειαν βάσιν ἔχωσιν [αἱ γωνίαι]. ὅπερ ἔδει δεῖξαι.



So let another (straight-line) have been inflected, and let there be another angle,  $BDC$ . And  $DE$  being joined, let it have been produced to  $G$ . So, similarly, we can show that angle  $GEC$  is double  $EDC$ , of which  $GEB$  is double  $EDB$ . Thus, the remaining (angle)  $BEC$  is double the (remaining angle)  $BDC$ .

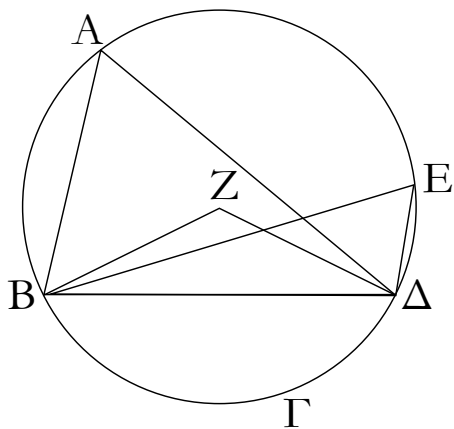
Thus, in a circle, the angle at the center is double that at the circumference, when [the angles] have the same circumference base. (Which is) the very thing it was required to show.

κα'.

Proposition 21

Ἐν κύκλῳ αἱ ἐν τῷ αὐτῷ τμήματι γωνίαι ἴσαι ἀλλήλαις εἰσίν.

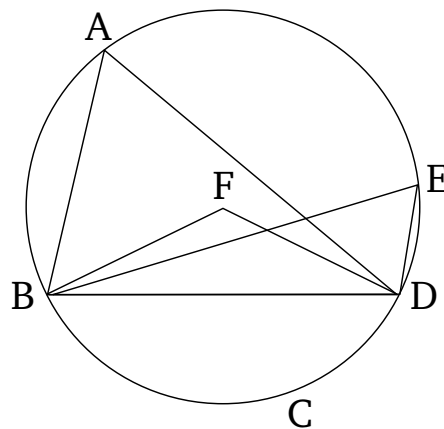
In a circle, angles in the same segment are equal to one another.



Ἐστω κύκλος ὁ  $AB\Gamma\Delta$ , καὶ ἐν τῷ αὐτῷ τμήματι τῶν  $BAE\Delta$  γωνίαι ἔστωσαν αἱ ὑπὸ  $BA\Delta$ ,  $BE\Delta$ . λέγω, ὅτι αἱ ὑπὸ  $BA\Delta$ ,  $BE\Delta$  γωνίαι ἴσαι ἀλλήλαις εἰσίν.

Εἰλήφθω γὰρ τοῦ  $AB\Gamma\Delta$  κύκλου τὸ κέντρον, καὶ ἔστω τὸ  $Z$ , καὶ ἐπεζεύχθωσαν αἱ  $BZ$ ,  $Z\Delta$ .

Καὶ ἐπεὶ ἡ μὲν ὑπὸ  $BZ\Delta$  γωνία πρὸς τῷ κέντρῳ ἐστίν, ἡ δὲ ὑπὸ  $BA\Delta$  πρὸς τῇ περιφερείᾳ, καὶ ἔχουσι τὴν αὐτὴν περιφέρειαν βάσιν τὴν  $B\Gamma\Delta$ , ἡ ἄρα ὑπὸ  $BZ\Delta$  γωνία διπλασίον ἐστὶ τῆς ὑπὸ  $BA\Delta$ . διὰ τὰ αὐτὰ δὴ ἡ ὑπὸ  $BZ\Delta$  καὶ τῆς ὑπὸ



Let  $ABCD$  be a circle, and let  $BAD$  and  $BED$  be angles in the same segment  $BAED$ . I say that angles  $BAD$  and  $BED$  are equal to one another.

For let the center of circle  $ABCD$  have been found [Prop. 3.1], and let it be (at point)  $F$ . And let  $BF$  and  $FD$  have been joined.

And since angle  $BFD$  is at the center, and  $BAD$  at the circumference, and they have the same circumference base  $BCD$ , angle  $BFD$  is thus double  $BAD$  [Prop. 3.20].

$BE\Delta$  ἐστὶ διπλοῦν· ἴση ἄρα ἢ ὑπὸ  $BA\Delta$  τῆ ὑπὸ  $BE\Delta$ .

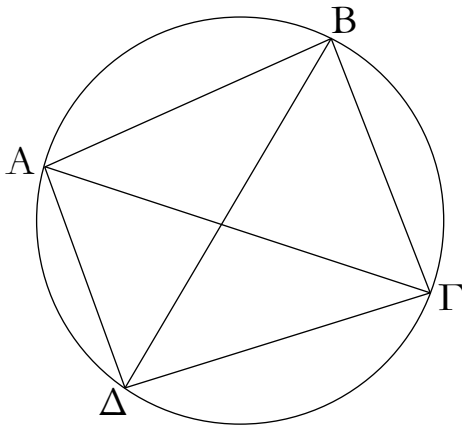
Ἐν κύκλῳ ἄρα αἱ ἐν τῷ αὐτῷ τμήματι γωνίαι ἴσαι ἀλλήλαις εἰσὶν· ὅπερ ἔδει δεῖξαι.

So, for the same (reasons),  $BFD$  is also double  $BED$ . Thus,  $BAD$  (is) equal to  $BED$ .

Thus, in a circle, angles in the same segment are equal to one another. (Which is) the very thing it was required to show.

κβ'.

Τῶν ἐν τοῖς κύκλοις τετραπλεύρων αἱ ἀπεναντίον γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσὶν.



Ἐστω κύκλος ὁ  $AB\Gamma\Delta$ , καὶ ἐν αὐτῷ τετράπλευρον ἔστω τὸ  $AB\Gamma\Delta$ . λέγω, ὅτι αἱ ἀπεναντίον γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσὶν.

Ἐπεζεύχθωσαν αἱ  $AG$ ,  $B\Delta$ .

Ἐπεὶ οὖν παντὸς τριγώνου αἱ τρεῖς γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσὶν, τοῦ  $AB\Gamma$  ἄρα τριγώνου αἱ τρεῖς γωνίαι αἱ ὑπὸ  $\Gamma AB$ ,  $AB\Gamma$ ,  $B\Gamma A$  δυσὶν ὀρθαῖς ἴσαι εἰσὶν. ἴση δὲ ἢ μὲν ὑπὸ  $\Gamma AB$  τῆ ὑπὸ  $B\Delta\Gamma$ · ἐν γὰρ τῷ αὐτῷ τμήματι εἰσι τῷ  $BA\Delta\Gamma$ · ἢ δὲ ὑπὸ  $\Gamma B\Delta$  τῆ ὑπὸ  $A\Delta B$ · ἐν γὰρ τῷ αὐτῷ τμήματι εἰσι τῷ  $A\Delta\Gamma B$ · ὅλη ἄρα ἢ ὑπὸ  $A\Delta\Gamma$  ταῖς ὑπὸ  $BAG$ ,  $\Gamma B\Delta$  ἴση ἐστίν. κοινὴ προσκείσθω ἢ ὑπὸ  $AB\Gamma$ · αἱ ἄρα ὑπὸ  $AB\Gamma$ ,  $BAG$ ,  $\Gamma B\Delta$  ταῖς ὑπὸ  $AB\Gamma$ ,  $A\Delta\Gamma$  ἴσαι εἰσὶν. ἀλλ' αἱ ὑπὸ  $AB\Gamma$ ,  $BAG$ ,  $\Gamma B\Delta$  δυσὶν ὀρθαῖς ἴσαι εἰσὶν. καὶ αἱ ὑπὸ  $AB\Gamma$ ,  $A\Delta\Gamma$  ἄρα δυσὶν ὀρθαῖς ἴσαι εἰσὶν. ὁμοίως δὲ δείξομεν, ὅτι καὶ αἱ ὑπὸ  $BA\Delta$ ,  $\Delta\Gamma B$  γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσὶν.

Τῶν ἄρα ἐν τοῖς κύκλοις τετραπλεύρων αἱ ἀπεναντίον γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσὶν· ὅπερ ἔδει δεῖξαι.

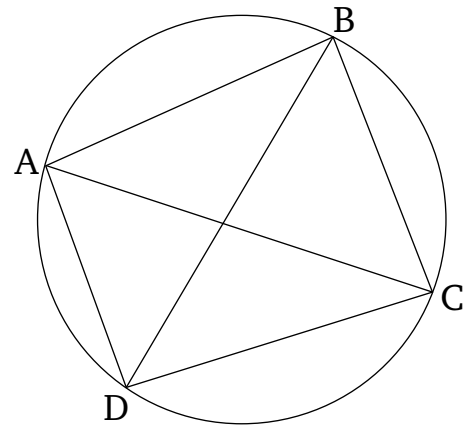
κγ'.

Ἐπὶ τῆς αὐτῆς εὐθείας δύο τμήματα κύκλων ὅμοια καὶ ἄνισα οὐ συσταθήσεται ἐπὶ τὰ αὐτὰ μέρη.

Εἰ γὰρ δυνατόν, ἐπὶ τῆς αὐτῆς εὐθείας τῆς  $AB$  δύο τμήματα κύκλων ὅμοια καὶ ἄνισα συνεστάτω ἐπὶ τὰ αὐτὰ μέρη τὰ  $\Gamma B$ ,  $A\Delta B$ , καὶ διήχθω ἢ  $AG\Delta$ , καὶ ἐπεζεύχθωσαν

### Proposition 22

For quadrilaterals within circles, the (sum of the) opposite angles is equal to two right-angles.



Let  $ABCD$  be a circle, and let  $ABCD$  be a quadrilateral within it. I say that the (sum of the) opposite angles is equal to two right-angles.

Let  $AC$  and  $BD$  have been joined.

Therefore, since the three angles of any triangle are equal to two right-angles [Prop. 1.32], the three angles  $CAB$ ,  $ABC$ , and  $BCA$  of triangle  $ABC$  are thus equal to two right-angles. And  $CAB$  (is) equal to  $BDC$ . For they are in the same segment  $BADC$  [Prop. 3.21]. And  $ACB$  (is equal) to  $ADB$ . For they are in the same segment  $ADCB$  [Prop. 3.21]. Thus, the whole of  $ADC$  is equal to  $BAC$  and  $ACB$ . Let  $ABC$  have been added to both. Thus,  $ABC$ ,  $BAC$ , and  $ACB$  are equal to  $ABC$  and  $ADC$ . But,  $ABC$ ,  $BAC$ , and  $ACB$  are equal to two right-angles. Thus,  $ABC$  and  $ADC$  are also equal to two right-angles. Similarly, we can show that angles  $BAD$  and  $DCB$  are also equal to two right-angles.

Thus, for quadrilaterals within circles, the (sum of the) opposite angles is equal to two right-angles. (Which is) the very thing it was required to show.

### Proposition 23

Two similar and unequal segments of circles cannot be constructed on the same side of the same straight-line.

For, if possible, let the two similar and unequal segments of circles,  $ACB$  and  $ADB$ , have been constructed on the same side of the same straight-line  $AB$ . And let