## Conics excercise

*a.* Let two points $A$ and $B$ be given, say with (cartesian) coordinates $A = (-\frac{a}{2}, 0)$ and $B = (\frac{a}{2}, 0)$. The point $C$ moves in the plane in such a way that the difference of the distances $CA$ and $CB$ remains constant, say $b$. Assuming $a > b$, prove that the coordinates $(x, y)$ of $C$ satisfy the equation

$$\frac{b^2 y^2}{a^2 - b^2} = x^2 - \frac{b^2}{4}.$$

Rewrite this equation in the form of one of the standard equations for conic sections: $y^2 = \alpha x$, $y^2 = (\alpha - \beta x)x$, or $y^2 = (\alpha + \beta x)x$. Which kind of conic section does $C$ trace out? *Hint: this is prop. 16 in De Witt's* Elementa Curvarum II*; you can glance over his proof and deduce what you have to do even without a knowledge of Latin. But be advised that he writes "⊃o" for "$=$", and "$x = a$" for "$|x - a|$", and that he uses $v = x - \frac{a}{2}$ instead your $x$.*

*b.* The following diagram is taken from Frans van Schooten's *Tractaet van de Tuygh-werckelycke Beschrijving der Kegel-sneden op een Vlack* (Treatise of the Mechanical Description of Conic Sections on a Plane). The straight arms of the instrument are allowed to rotate around the fixed points $C$ and $F$. Further we have $CE = FK$, $CD = FG = EK$, and $DG = CF$. Show that the hand with pencil at $\varepsilon$ traces out a branch of a hyperbola.