f scalarveld, $\bar{F}$ vectorveld in $\mathbb{R}^3$

$n =$	78
$-$	44
o	25
$+$	9

$$\text{grad } f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) f = \nabla f$$

met $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

NIEUW:

$$\text{div } \bar{F} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot \bar{F} = \nabla \cdot \bar{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

met $\bar{F} = (F_1, F_2, F_3)$

divergentie

NIEUW

$$\text{curl } \bar{F} = \text{rot } \bar{F} = \nabla \times \bar{F} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)$$

$\downarrow$
rotation

Voorbeeld: $\bar{F} = (x, y, z)$

$$\operatorname{div} \bar{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = 3$$

$$\operatorname{curl} \bar{F} = (0, 0, 0) = \bar{0}$$

<u>Merk op</u>: als $\bar{F}$ conservatief is dan bestaat er een potentiaal f met eigenschap $\operatorname{grad} f = \bar{F}$

dus
$$\begin{aligned}
\operatorname{curl} \bar{F} &= \operatorname{curl}(\operatorname{grad} f) \\
&= \operatorname{curl}\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) \\
&= \left(\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y}, \frac{\partial^2 f}{\partial z \partial x} - \frac{\partial^2 f}{\partial x \partial z}, \frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x}\right) \\
&= (0, 0, 0) = \bar{0}
\end{aligned}$$

Belangrik: curl grad $f = \bar{0}$

$$\bar{F} = (0, x, 0) = x\hat{j}$$

$$\text{curl } \bar{F} = (0, 0, 1) \qquad \text{div curl } \bar{F} = \text{div}(0,0,1) = 0$$

Voor will. gladde vectorveld $\bar{F} = (F_1, F_2, F_3)$

$$\nabla \times \bar{F} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right)$$

$$\nabla \cdot (\nabla \times \bar{F}) = \cancel{\frac{\partial^2 F_3}{\partial x \partial y}} - \cancel{\frac{\partial^2 F_2}{\partial x \partial z}} + \cancel{\frac{\partial^2 F_1}{\partial y \partial z}} - \cancel{\frac{\partial^2 F_3}{\partial y \partial x}} + \cancel{\frac{\partial^2 F_2}{\partial z \partial x}} - \cancel{\frac{\partial^2 F_1}{\partial z \partial y}}$$

$$= 0$$

DIT IS OOK BELANGRYK: div curl $\bar{F} = 0$

NB:

grad f = vector ∇f

curl $\bar{F}$ = vector $\nabla \times \bar{F}$

div $\bar{F}$ = scalar $\nabla \cdot \bar{F}$

div grad f = scalar

$$= \nabla \cdot \nabla f = \nabla \cdot \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$= \Delta f$ Laplaciaan van f

$= \nabla^2 f$ (notatie)

Zie boek vector identities p. 915

Zoals bijv.

a) $\nabla(fg) = f\nabla g + g\nabla f$ etc

Weten: $\operatorname{div} \operatorname{curl} \bar{F} = 0$ en $\operatorname{curl} \operatorname{grad} f = \bar{0}$

De rest niet uit je hoofd leren

Wel kunnen "bewijzen" door uitschrijven in coord.

Handig schema:

$$\boxed{f \xrightarrow{\text{grad}} F \xrightarrow{\text{curl}} G \xrightarrow{\text{div}} g}$$

$$f \xrightarrow{\text{grad}} \bar{F}$$

$$\bar{F} \xrightarrow{\text{curl}} \bar{G}$$

$$\bar{G} \xrightarrow{\text{div}} g$$

Nota bene:

2 stapjes achter elkaar $= 0$ of $\bar{0}$